

Resin Matrix Shear Band and Its Special Effect on Stress Concentration of Fiber Composites^{*}

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Abstract A constitutive model is constructed to consider the resin matrix post-yield softening and progressive hardening behaviors. A user-defined material mechanical behavior (UMAT) subroutine is created, then the non-linear three-dimensional finite element analysis on the tensile processes of multi-fiber composites is conducted. The approximate 45° shear bands emanating from the matrix crack tip are found, being coincided with the experimental observations. The shear stress on the adjacent intact fiber/matrix interface is strongly influenced by the shear band and thus the stress concentration factor (SCF) changes obviously in the adjacent fibers. The distinct stress redistribution in the adjacent intact fibers implies the significant effect of the shear bands on the progressive fiber fracture initiation. As the inter-fiber spacing increases, the peak value of the SCF in the adjacent intact fiber decreases, whereas the overload zone becomes wider. The research has provided a helpful tool to evaluate the failure of fiber composites and optimize the composite performance through the proper selection of resin matrix properties and fiber volume fraction.

Keywords: Fiber composite; Stress concentration; Shear band; Fiber volume fraction; Theory and modeling.

INTRODUCTION

The stress concentration around a fiber breakage is widely regarded as a key parameter controlling the strength and failure behavior of composites^[1]. It may lead to the fracture of the adjacent fiber, ultimately resulting in the catastrophic failure of composite structures. A number of researchers have extensively studied the stress concentration phenomenon experimentally, theoretically and numerically^[2–7]. In unidirectional fiber composites, the additional tensile stress in the adjacent intact fiber is induced by the surrounding shear stress at the fiber/matrix interface. When the interface is intact, the overload is redistributed among the adjacent fibers through the matrix shear deformation. Therefore, the matrix plays a significant role in controlling the stress concentration as it influences the way in which the load is transferred from the broken fiber to its neighboring fibers^[8]. The importance of the matrix plasticity was also indicated in experimental reports^[9, 10], after which considerable attentions have been paid to the matrix plasticity. However, the theoretical models predicting the stress redistribution either neglect the matrix properties, or are proposed only for elastic/elasto-plastic matrices^[11, 12]. The softening after yield and progressive hardening behaviors of glassy matrix have not been comprehensively investigated to date.

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Furthermore, the stress concentration depends not only on the stress-strain relationship of the matrix but also on the fiber volume fraction (V_f), which in turn depends upon the inter-fiber distance ($d_{\text{inter-fiber}}$)^[7, 13]. From a micromechanics perspective, it is necessary to provide a means of probing the interaction between fibers. Most of the research has focused on the laser Raman spectroscopy (LRS) as a detection tool for directly measuring the strain in the broken and unbroken fibers^[6, 14]. However, the applicability of the LRS in the glass fiber composites is very limited so far since this technique is restricted to the Raman-active fibers. An alternative way to investigate the complicated fiber-fiber-interaction effect is to use a numerical analysis^[15], which is based on the continuum mechanics and can provide useful insight into the behavior of a material at microscopic level^[16].

In the present work, the non-linear finite element (FE) simulation of the tensile process of three-dimensional (3D) multi-fiber composites was conducted, attempting to picture the deformation pattern of the glassy matrix around a fiber breakage. A proper constitutive model was constructed to consider the matrix softening after yield and progressive hardening behaviors. Then the occurrence of a shear band and its effect on the stress concentration factor (SCF)^[9] in the adjacent fibers were investigated in detail.

MATERIALS AND METHODS

Geometric Model

ABAQUS software was used to carry out the 3D elasto-plastic FE analysis for a unidirectional fiber reinforced composite with a fiber breakage. Due to the hexagonal fiber array and the geometric symmetry, only 1/8 of the composite material model was explicitly constructed. As shown in Fig. 1, the constructed model included a quarter broken fiber in the center, one and a half adjacent intact fibers, as well as the surrounding matrix. An area of equivalent homogeneous composite was added to the model, accounting for the influence of the surrounding composite on the mechanical performance^[17]. For the entire model, both the width (X-direction) and the height (Y-direction) were 50 μm , while the length (Z-direction) was 75 μm to ensure that the opposite face was far away from the broken face, so that the central fiber fracture could not disturb the uni-axial stress state on the opposite face. In order to investigate the influence of matrix plasticity on the SCF mainly, the fiber and matrix were assumed to be well bonded so that the debonding was not permitted^[18]. Furthermore, up to 95925 6-node linear triangular prism elements were used to construct the mesh in order to get accurate results at the key area. All simulations were carried out at a workgroup with Intel® Xeon® 24 CPU E5645 (2.4GHz), 32GB of memory, under Microsoft Windows XP Professional X64 edition.

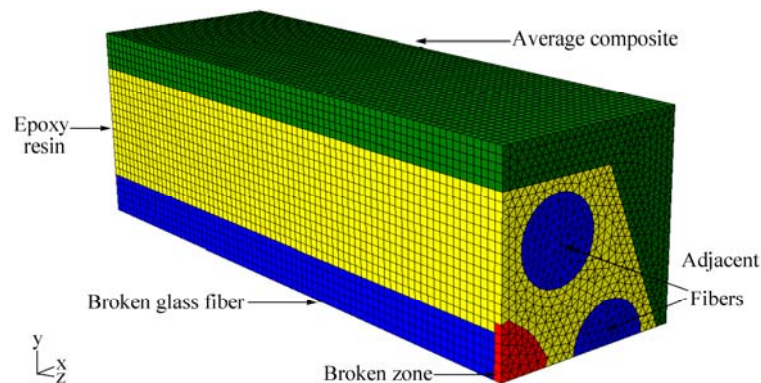


Fig. 1 Schematic diagram of finite element model for a continuous glass fiber reinforced epoxy composite

To examine the influence of the matrix softening under different $d_{\text{inter-fiber}}$, four values of $d_{\text{inter-fiber}}$ were chosen, namely $1.1d$ (11 μm), $1.3d$ (13 μm), $1.5d$ (15 μm) and $2d$ (20 μm), where d denotes the fiber diameter (10 μm). For the hexagonal fiber array, the corresponding V_f is obtained by:

$$V_f = \frac{\pi}{2\sqrt{3}} \left(\frac{d}{d_{\text{inter-fiber}}} \right)^2 \quad (1)$$

Boundary Conditions

The tensile load was applied to the meso-scopic composite model by prescribing a Z-displacement to the nodes at the far end of the model. A total strain of 2% with respect to the initial Z-length was applied. A “pre-break” was introduced by applying no constraints to the nodes at the broken end of the center fiber and the surrounding crack zone in matrix, while the Z-direction displacements of the rest nodes at $Z = 0$ were equal to zero owing to symmetry. To meet the symmetry, throughout the analysis the nodes at $X = 0$ were fixed in the X-direction, and the nodes at $Y = 0$ were constrained in the Y-direction.

Constitutive Models

In the present study we follow the modified BPA model given by Wu and van der Giessen^[19], to take into account the intrinsic softening and progressive strain hardening behavior of polymer matrix. Neglecting thermal effects and assuming the elastic strains to remain small^[20], the rate of deformation $\mathbf{D}$ is composed of an elastic part and a plastic part: $\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p$, while the latter item $\mathbf{D}^p$ is specified through the equation:

$$\mathbf{D}^p = \frac{\dot{\gamma}^p}{\sqrt{2}\tau} \bar{\boldsymbol{\sigma}}' \quad (2)$$

where $\bar{\boldsymbol{\sigma}}'$ is the deviatoric part of the driving stress, and τ is the equivalent shear stress

$$\tau = \sqrt{\frac{1}{2} \bar{\boldsymbol{\sigma}}' \cdot \bar{\boldsymbol{\sigma}}'} \quad (3)$$

The equivalent plastic shear strain rate $\dot{\gamma}^p$ is given by

$$\dot{\gamma}^p = \dot{\gamma}_0 \exp\left[-\frac{A\tilde{s}}{T} \left(1 - \left(\frac{\tau}{\tilde{s}}\right)^{5/6}\right)\right] \quad (4)$$

where $\dot{\gamma}_0$ and A are material parameters, and T is the absolute temperature which is taken constant in the present study. The shear resistance $\tilde{s}$ is assumed to evolve with plastic straining via

$$\tilde{s}(\gamma^p) = s_s + (s_0 - s_s) \exp(-h\gamma^p) + \alpha p \quad (5)$$

from an initial value s_0 to a saturation value s_s in order to model the intrinsic softening following the pronounced yield point, p is the pressure, while α and h are the additional material parameters.

The progressive hardening after yield is described by the back stress tensor $\mathbf{b}$ incorporated in the driving stress tensor $\bar{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - \mathbf{b}$. Here, we shall combine the three-chain and eight-chain model via

$$\mathbf{b} = (1 - \rho) \mathbf{b}^{3\text{-ch}} + \rho \mathbf{b}^{8\text{-ch}} \quad (6)$$

where ρ is determined by the maximum principal plastic stretch, $\mathbf{b}^{3\text{-ch}}$ denotes the three-chain model^[21], $\mathbf{b}^{8\text{-ch}}$ denotes the eight-chain model^[22]. Two additional material parameters are introduced to calculate the back stress tensor: the initial hardening modulus C_R and the chain limit stretch λ_{max} .

As to the constitutive model for the reinforcing fiber, a linear elastic relationship is employed over the entire range of loading, so is the average composite, because of the very low yield stress of the matrix and the high strength and stiffness of the fiber.

Material Parameters

In order to capture the shear band clearly by means of polarized light microscope and then compare the experimental results with the simulated results, the composite under investigation is a glass fiber/epoxy system. The detailed parameters for epoxy DER 332^[23, 24] are listed in Table 1, with the character of immediate softening

after yield and progressive hardening. The inputted parameters are conveniently defined in the ABAQUS code through the user defined material subroutine (UMAT). The following parameters for a typical glass fiber are used to describe the reinforcing material: Young's modulus $E = 72$ GPa, and Poisson's ratio $\nu = 0.22$. For the mechanical properties of the average composite part of the model, an isotropic behavior is assumed. Being estimated using the rule of mixture^[25], the mechanical properties of the average composite vary with $d_{\text{inter-fiber}}$.

Table 1. Material properties of the epoxy resin used in UMAT

E (MPa)	ν	s_0 (MPa)	s_s/s_0	As_0/T	h/s_0	α	λ_{max}	C_R/s_0	γ_0 (s ⁻¹)
2500	0.3	96.7	0.95	78.955	17.25	0.08	2.5	0.059	2×10^{15}

RESULTS AND DISCUSSION

Matrix Deformation Pattern

Without loss of generality, we first take a look at the deformation pattern of the composite with $d_{\text{inter-fiber}} = 2d$. According to Eq. (1), the corresponding $V_f = 22.68\%$. The deformation pattern of the composite visualized by the distribution of the Z-direction logarithmic strain in YOZ plane at the total strain of 2% is shown in Fig. 2(b), from which it can be easily seen that two pairs of intensely deformed zones have formed in the matrix, approximately 45° starting from the matrix crack tip. The shape, location and orientation of these narrow zones agree well with that of the shear bands observed in our experiment (Fig. 2a).

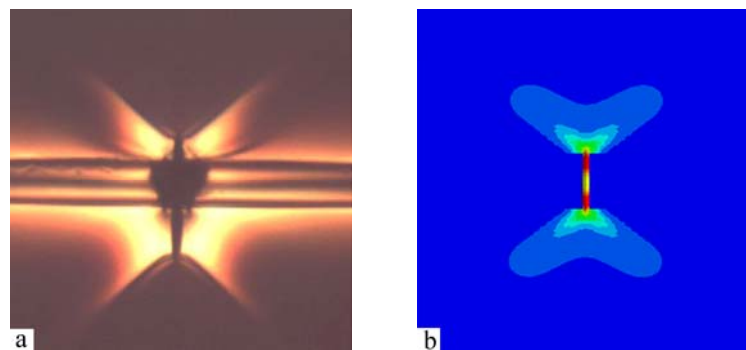


Fig. 2 Comparison between the simulated result and the experimental observation: (a) polarized optical micrograph of shear bands captured in the glass fiber/epoxy composite tensile test and (b) the Z-direction logarithmic strain contour in YOZ plane of the composite model

The contour of Z-direction displacement in YOZ plane is presented in Fig. 3(a). There is an obvious displacement gradient in the matrix near the fiber breakage along the Z-direction, although the displacement gradient in the farther matrix is more uniform. The simulated deformation pattern is identical with the one in Fig. 2(b), which identifies that it is the Z-direction strain that makes the major contribution to the highly intense deformation. During the tensile process, the materials on about 45° long sharp areas are moving away from each other along the Z-direction, leaving the intensely stretched narrow zones. The statement is immediately indicated by Fig. 3(b) illustrating the directions towards which the materials are being stretched at the strain of 2%. It is not hard to infer that once the overall strain is large enough, the materials on these severely deformed areas are likely to be pulled apart, leading to 45° matrix cracks, as reported in Fig. 3 of Ref. [26]. Taking all the above discussions into consideration, we are convinced that the simulated patterns shown in Fig. 2(b) are exactly what we meant by shear bands in Fig. 2(a).

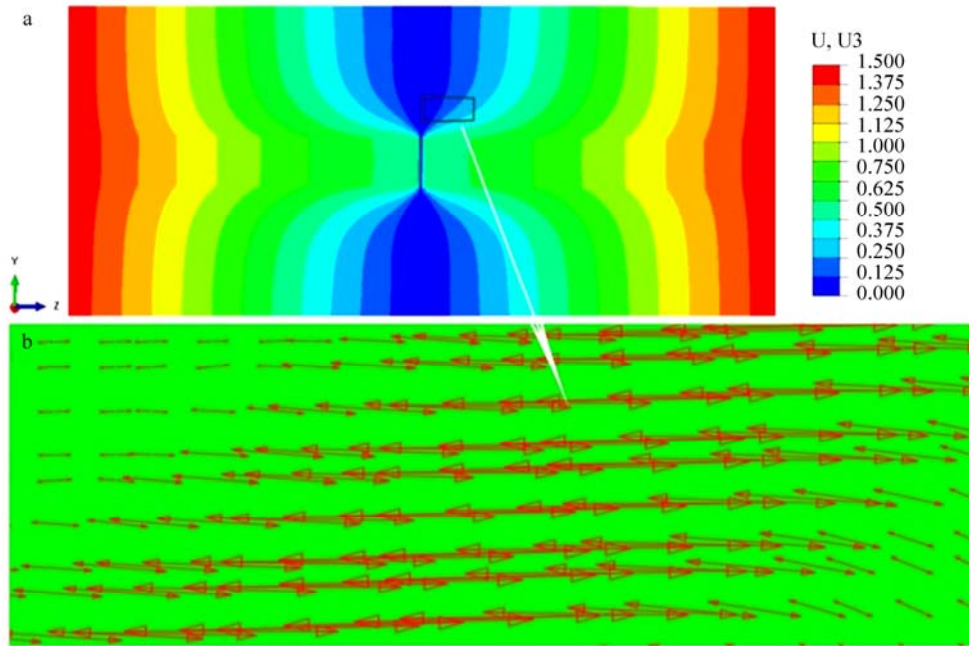


Fig. 3 Contour of the deformation in YOZ plane at the strain of 2%: (a) Z-direction displacement; (b) orientation of the maximum principal logarithmic strain of the matrix within a rectangle (marked in Fig. 3a) The size of the arrows represents the relative magnitude of the strain.

Influence of Shear Bands on the Shear Stress at the Interface

As is well known, the mismatch between the modulus of the fiber and matrix contributes to the shear stress on the interface^[27], and the magnitude of the shear stress is highly affected by the matrix properties and inter-fiber spacing^[28, 29]. Figure 4 shows the distributions of the shear stress at different interfaces at an applied strain of 2%. Obviously, the appearance of breakage in the model induces an obvious change in the local shear stress around it. As can be seen from Fig. 4(a), because of the occurrence of a fiber-break in the unidirectional fiber composite, a large fraction of the load carried by the broken fiber is transferred to the matrix through the broken fiber/matrix interface^[30, 31], which results in a high shear stress at the interface between the broken fiber and the matrix. It should be noted that all the maximum shear stresses are located at $Z = 0$.

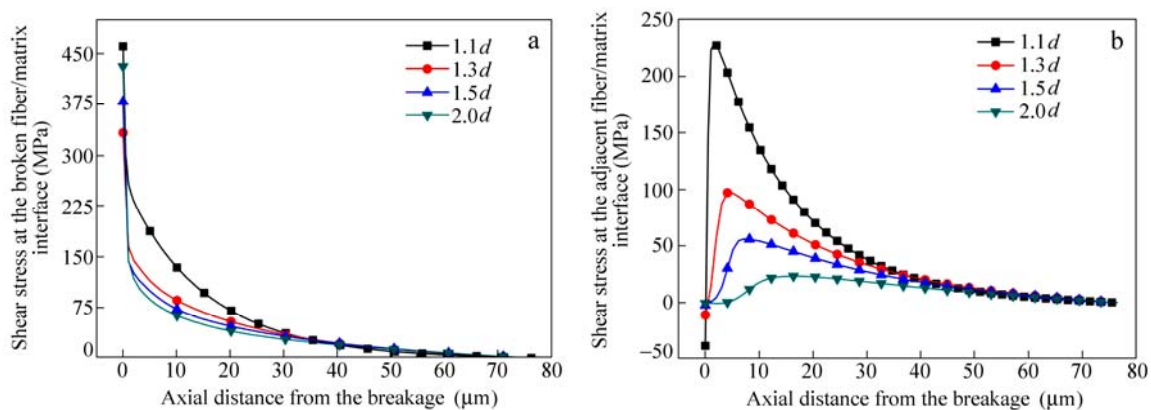


Fig. 4 Distributions of the shear stress at an applied strain of 2% and over a range of inter-fiber spacings: (a) at the broken fiber/matrix interface and (b) at the adjacent intact fiber/matrix interface

Meanwhile, the distribution of the shear stress along the interface between the adjacent intact fiber and matrix is shown in Fig. 4(b). It can be seen that the appearance of the shear band results in a considerable non-uniform distribution of shear stress. A local maximum is found on the distribution curve, after which the shear stress decreases smoothly to zero along the Z-direction. What is interesting is that all the maximum shear stresses are not located at the starting point of the bond where $Z = 0$, which indicates that the shear stress transferred from the matrix to the adjacent intact fiber is affected by the local shear band in the matrix. The shift of the maximum shear stress away from the fiber end can be understood by looking at the directions towards which the matrix is stretched in Fig. 3(b). On the adjacent intact fiber/matrix interface, the part of matrix that is located at the front end of the shear band is severely pulled away along the Z-direction. As a result of this, the maximum shear stress exists in the serious deformed areas. Furthermore, it can be inferred that the high shear stress front edge, where the maximum deformations occur, is likely to be the zone where comes the stress concentration.

Influence of Shear Bands on the Stress Redistribution at the Adjacent Intact Fiber

Through a series of comparative studies^[32–34], we conclude that the post-yield softening character determines the development of the shear band, which can further disturb the stress state of the adjacent fibers. To describe conveniently the overloading of an intact fiber as a result of a broken fiber, a stress concentration factor (SCF) is defined as the ratio of the maximum stress in the intact fiber to the stress in the undisturbed part of the intact fiber^[9]. Figure 5 presents the SCF distribution in the adjacent intact fiber with different $d_{\text{inter-fiber}}$. It can be seen that the introduction of a fiber-break results in a considerable increase of the SCF at the immediate vicinity of the break. However, when the shear bands are fully developed due to the matrix softening, two peaks appear at the edges of the overstress zone, accompanying with the stress decreasing dramatically along the Z-direction. Further examination finds that the positions of the two additional stress concentration peaks coincide well with the front ends of the two shear bands. It clearly demonstrates that the shear bands could extend the overstress zone in the adjacent fibers, by inducing the stress concentration when the front ends of the shear bands reach the adjacent fibers.

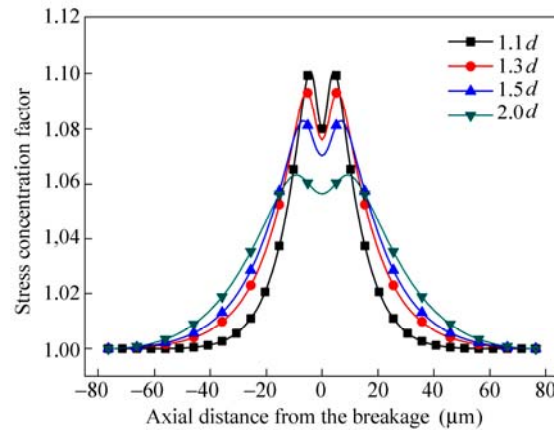


Fig. 5 Distributions of the stress concentration factor in the adjacent intact fiber

In addition, a narrower zone undergoes much higher stress for the composites with the matrix exhibiting the post-yield softening under a smaller $d_{\text{inter-fiber}}$ (e.g. $1.1d$, $V_f = 74.96\%$), which means that the yielding matrix is extremely inclined to constrain the subsequent breakages on the adjacent intact fibers within a narrower zone. However, what should be pointed out is, even under the condition of a small inter-fiber spacing, the fiber breakages are not vertically aligned, as revealed by the shape of the SCF curves (for $1.1d$ cases) in Fig. 5. The peak value appears at the position where $Z = 4.06 \mu\text{m}$.

It can also be noted that the maximum SCF decreases with the increase of $d_{\text{inter-fiber}}$. For example, when $d_{\text{inter-fiber}}$ increases from $1.1d$ to $2.0d$, the corresponding V_f decreases from 74.96% to 22.68% , and the maximum SCF changes from 1.10 to 1.06 . It is due to the fact that more energy released from the broken fiber is absorbed

by the increasing volume of the matrix, which undergoes the shear yield and softening. Thus, the stress transfer capacity of the matrix is reduced accordingly, and then a lower SCF is redistributed in the adjacent intact fiber. This is also associated with the decrease in the maximum shear stress as shown in Fig. 4(b).

Furthermore, the position where the peak value appears becomes farther from the breakage with the increase of $d_{\text{inter-fiber}}$. As shown in Fig. 5, the maximum SCF is located at $Z = 4.06 \mu\text{m}$ and $Z = 9.20 \mu\text{m}$ for the $1.1d$ and $2.0d$ -model, respectively. It is because the shear band extending to the adjacent intact fiber needs a bigger displacement when $d_{\text{inter-fiber}}$ becomes wider, which results in a larger Z -direction distance corresponding to the maximum SCF.

CONCLUSIONS

The role of the matrix shear band in the mechanical performance of the fiber-reinforced polymer matrix composite has been investigated using the non-linear 3D FE simulation. The intrinsic character of “post-yield softening” of glassy polymer makes it possible to promote the shear bands emanating from the tips of the matrix crack, which can not be displayed in the simulations using linear elastic or general elasto-plastic models for matrices. The shear stress on the adjacent intact fiber/matrix interface is strongly influenced by the shear band of the matrix. The way in which the load is transferred from the broken fiber to its neighboring fibers is changed, so is the SCF distribution in the adjacent fibers. The maximum SCFs shift away from the position where $Z = 0$ in case of the shear band existing. Moreover, the overstress zone in the adjacent fiber becomes wider with the increasing inter-fiber spacing, but the peak value of the SCF decreases. The present work has revealed the significance of the matrix shear bands and the inter-fiber spacing in controlling the fiber-fiber interaction.

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